

Honors Geometry Chapter 4 Packet

4-1 Classifying Triangles

- Identify and classify triangles by angles and sides.

4-2 Angles of Triangles

- Apply the Angle Sum Theorem.
- Apply the Exterior Angle Theorem.

4-3 Congruent Triangles

- Name and label corresponding parts of congruent triangles.
- Identify congruence transformations.

4-4 Proving Congruence- SSS, SAS

- Use the SSS Postulate to test for triangle congruence.
- Use the SAS Postulate to test for triangle congruence.

4-5 Proving Congruence- ASA, AAS

- Use the ASA Postulate to test for triangle congruence.
- Use the AAS Theorem to test for triangle congruence.
- Use the HL-Postulate to test for right triangle congruence.

4-6 Isosceles Triangles

- Use properties of isosceles triangles.
- Use properties of equilateral triangles.

4-7 Triangles and Coordinate Proof

- Position and label triangles for use in coordinate proofs.
- Write coordinate proofs.

***Remember all postulates, theorems, and formulas from prior chapters.**

4-1 Classifying Triangles

Classifying Triangles by Angles

Acute

Right

Obtuse

Equiangular

Classifying Triangles by Sides

Scalene

Isosceles

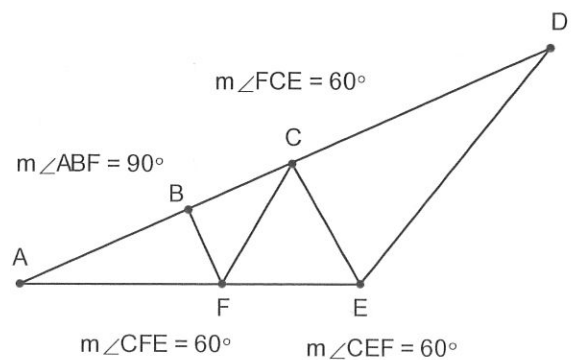
Equilateral

Answer with **always, sometimes or never.**

An isosceles triangle is an equilateral triangle.

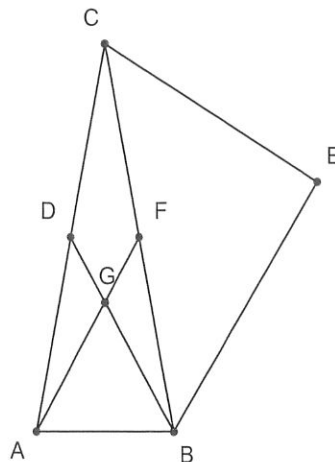
An equilateral triangle is an isosceles triangle.

Ex 1: The triangular truss below is modeled for steel construction. Classify $\triangle ADE$, $\triangle ABF$, and $\triangle FCE$ by angles.



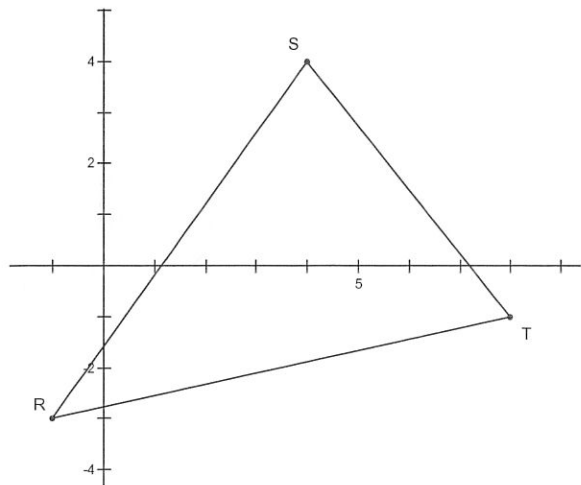
Is $\overline{BF} \parallel \overline{CE}$? Justify your answer.

Ex 2: Identify each triangle in the figure by sides if $\overline{CE} \perp \overline{BE}$, $\overline{AG} \cong \overline{GB}$, $\overline{AC} \cong \overline{CB}$.



Ex 3: Find d and the measure of each side of the equilateral $\triangle KLM$ if $KL = d + 2$, $LM = 12 - d$, and $KM = 4d - 13$.

Ex 4: Find the measures of the sides of $\triangle RST$. Classify the triangle by sides.



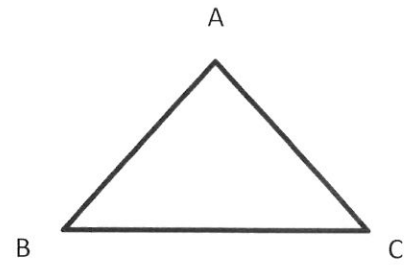
Turn to **page 184** and complete the Geometry Activity. Answer the corresponding questions below.

- | | |
|----|-----|
| 1. | 7. |
| 2. | |
| 3. | 12. |
| 4. | |
| 5. | |
| 6. | |

HW: pgs. 181-182 #28-31, 38, 40

4-2 Angles of Triangles

Angle Sum Theorem (4.1):

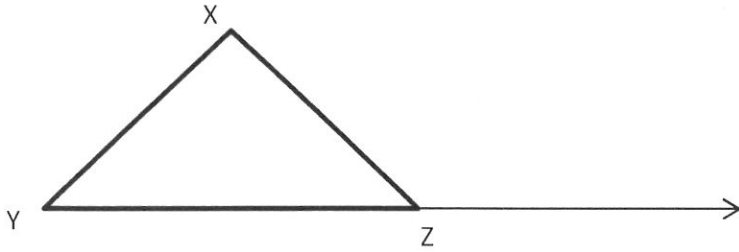


Proof of the Angle Sum Theorem

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.

Third Angle Theorem (4.2):

Exterior Angle:



Remote Interior Angles:

Exterior Angle Theorem (4.3):

Corollary:

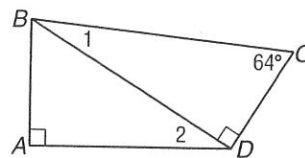
Corollary 4.1:

Corollary 4.2:

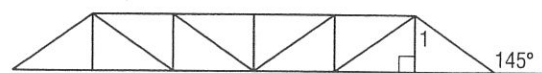
1. Find the measure of each angle if $\angle BAD$ and $\angle BDC$ are right angles and $m\angle ABC = 84^\circ$.

$$m\angle 1$$

$$m\angle 2$$



2. **CONSTRUCTION** The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find $m\angle 1$.

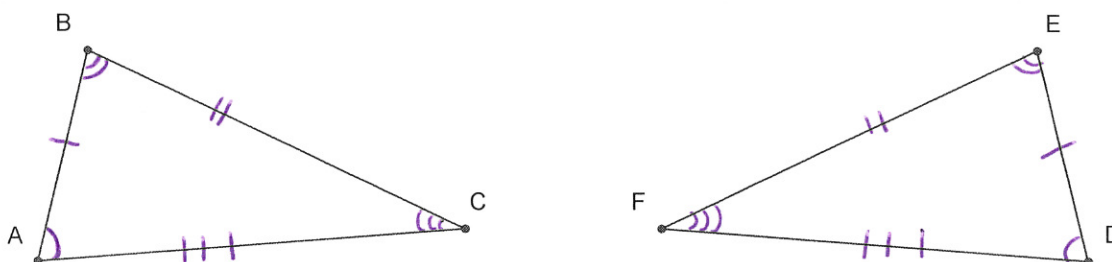


4-3 Congruent triangles

Congruent Triangles:

Definition of congruent Triangles (CPCTC):

CPCTC stands for:



$$\triangle ABC \cong \triangle DEF$$

* The corresponding vertices can be used to name the corresponding congruent sides and angles of the two triangles.

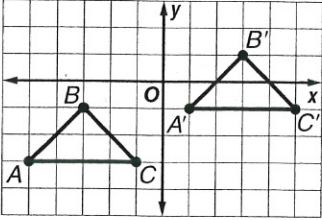
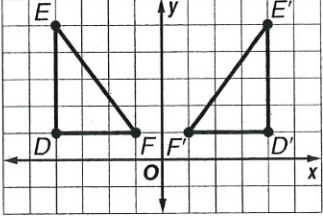
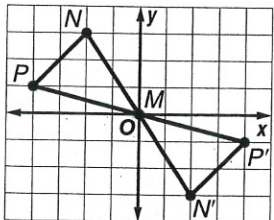
$$\begin{aligned} \angle A &\cong \angle D, \quad \angle B \cong \angle E, \quad \angle C \cong \angle F \\ \overline{AB} &\cong \overline{DE}, \quad \overline{BC} \cong \overline{EF}, \quad \overline{CA} \cong \overline{FD} \end{aligned}$$

* Congruent sides are opposite congruent angles.

Thm 4.4: Congruence of triangles is also reflexive symmetric, and transitive.

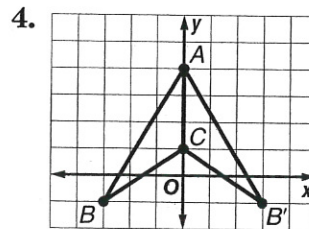
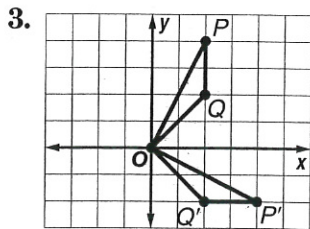
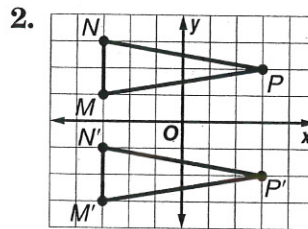
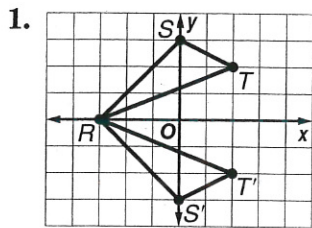
Reflexive	Symmetric	Transitive

Congruence Transformations:

Slide	Flip	Turn
$\triangle ABC \cong \triangle A'B'C'$ 	$\triangle DEF \cong \triangle D'E'F'$ 	$\triangle MNP \cong \triangle M'N'P'$ 

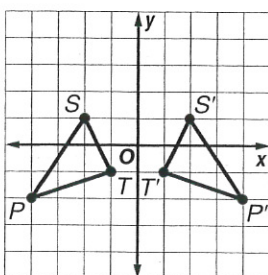
Exercises

Describe the congruence transformation between the two triangles as a *slide*, a *flip*, or a *turn*. Then name the congruent triangles.

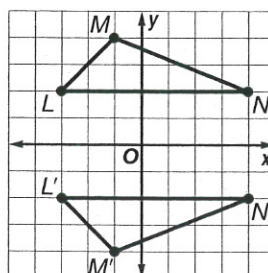


Verify that each of the following transformations preserves congruence, and name the congruence transformation.

5. $\triangle PST \cong \triangle P'S'T'$



6. $\triangle LMN \cong \triangle L'M'N'$



4-4 Proving Congruence- SSS, SAS

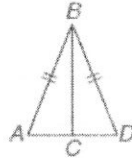
Warm-up: Currently, how can we prove triangles congruent?

SSS Postulate:

Example Write a two-column proof.

Given: $\overline{AB} \cong \overline{DB}$ and C is the midpoint of \overline{AD} .

Prove: $\triangle ABC \cong \triangle DBC$

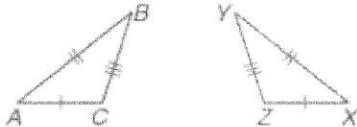


Statement	Reason

Exercises

Write a two-column proof.

1.

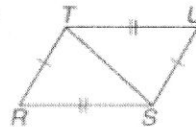


Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\overline{BC} \cong \overline{YZ}$

Prove: $\triangle ABC \cong \triangle XYZ$

Statements	Reasons

2.



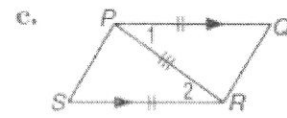
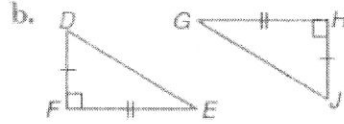
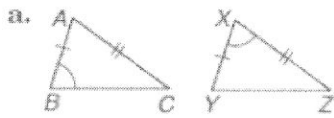
Given: $\overline{RS} \cong \overline{UT}$, $\overline{RT} \cong \overline{US}$

Prove: $\triangle RST \cong \triangle UTS$

Statements	Reasons

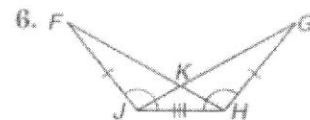
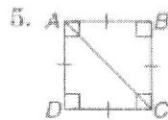
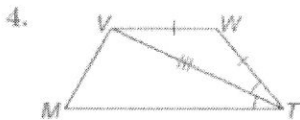
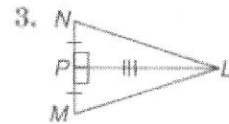
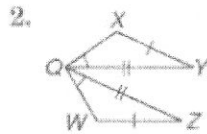
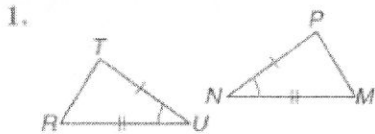
SAS Postulate:

Example For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.



Exercises

For each figure, determine which pairs of triangles can be proved congruent by the SAS Postulate.



4-4

Skills Practice

Proving Congruence—SSS, SAS

Determine whether $\triangle ABC \cong \triangle KLM$ given the coordinates of the vertices. Explain.

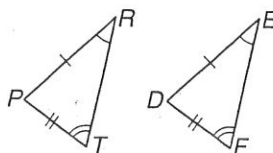
1. $A(-3, 3), B(-1, 3), C(-3, 1), K(1, 4), L(3, 4), M(1, 6)$

2. $A(-4, -2), B(-4, 1), C(-1, -1), K(0, -2), L(0, 1), M(4, 1)$

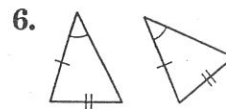
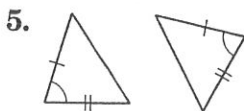
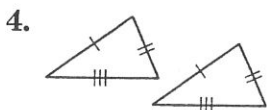
3. Write a ^{two column} flow proof.

Given: $\overline{PR} \cong \overline{DE}, \overline{PT} \cong \overline{DF}$
 $\angle R \cong \angle E, \angle T \cong \angle F$

Prove: $\triangle PRT \cong \triangle DEF$



Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.



Lesson 4-4

4-4 Practice

Proving Congruence—SSS, SAS

Determine whether $\triangle DEF \cong \triangle PQR$ given the coordinates of the vertices. Explain.

1. $D(-6, 1), E(1, 2), F(-1, -4), P(0, 5), Q(7, 6), R(5, 0)$

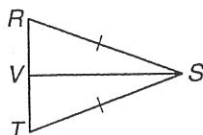
2. $D(-7, -3), E(-4, -1), F(-2, -5), P(2, -2), Q(5, -4), R(0, -5)$

3. Write a ~~flow~~ ^{two column} proof.

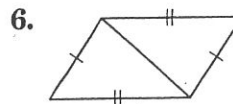
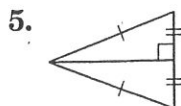
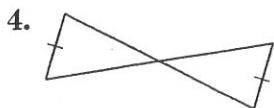
Given: $\overline{RS} \cong \overline{TS}$

V is the midpoint of \overline{RT} .

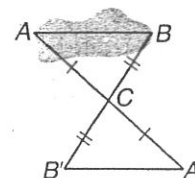
Prove: $\triangle RSV \cong \triangle TSV$



Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.



7. **INDIRECT MEASUREMENT** To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths $A'B'$ and AB are equal?



4-5 Proving Congruence – ASA, AAS

Included side:

Angle-Side-Angle Congruence Postulate (ASA post):

Angle-Angle-Side Congruence Theorem (AAS thm):

Proof of AAS Theorem:

Given:

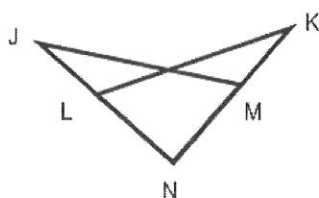
Proof:

Statement	Reason
1.	1.
2.	2.
3.	3.

Write a proof:

Given: $\angle NKL \cong \angle NJM, \overline{KL} \cong \overline{JM}$

Prove: $\overline{LN} \cong \overline{MN}$



Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

4-5 Practice

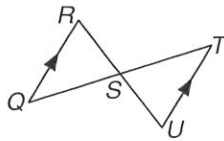
Proving Congruence—ASA, AAS

1. Write a two column proof.

Given: S is the midpoint of \overline{QT} .

$\overline{QR} \parallel \overline{TU}$

Prove: $\triangle QSR \cong \triangle TSU$

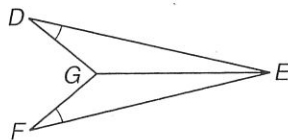


2. Write a two column proof.

Given: $\angle D \cong \angle F$

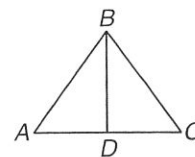
\overline{GE} bisects $\angle DEF$.

Prove: $\overline{DG} \cong \overline{FG}$



ARCHITECTURE For Exercises 3 and 4, use the following information.

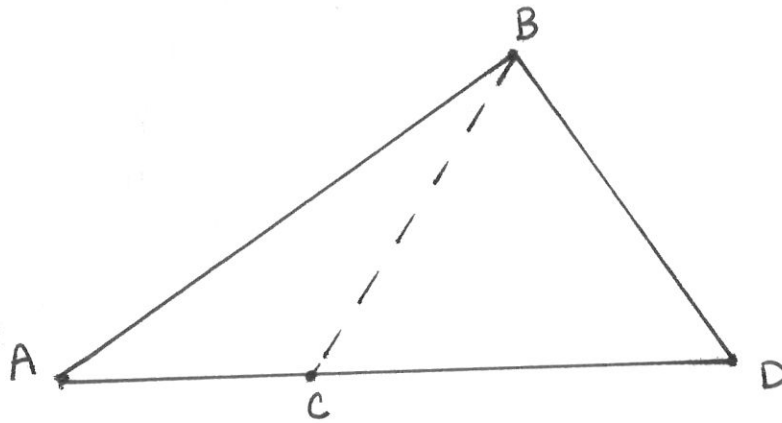
An architect used the window design in the diagram when remodeling an art studio. \overline{AB} and \overline{CB} each measure 3 feet.



3. Suppose D is the midpoint of \overline{AC} . Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

4. Suppose $\angle A \cong \angle C$. Determine whether $\triangle ABD \cong \triangle CBD$. Justify your answer.

4-5 Proving Congruence- ASS



Given: $\overline{BC} \cong \overline{BD}$

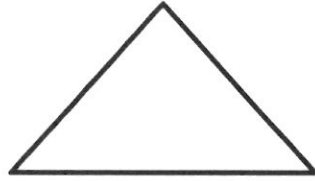
Prove: $\triangle ABC \cong \triangle CBD$

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.

ASS Congruence Postulate:

4-6 Isosceles Triangles

Vertex Angle:

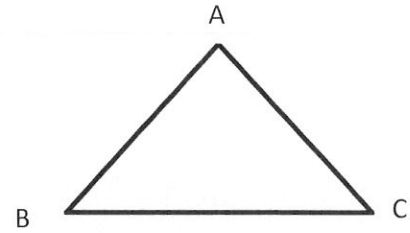


Base Angles:

Isosceles Triangle Theorem (4.9):

Proof of the Isosceles Triangle Theorem

Given: $\overline{AB} \cong \overline{AC}$



Prove:

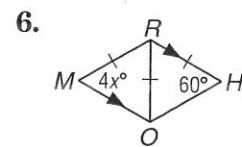
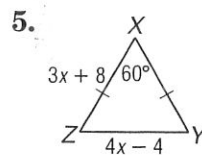
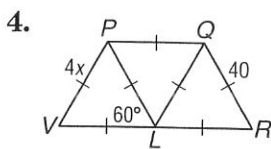
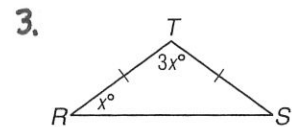
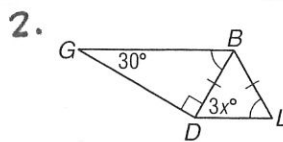
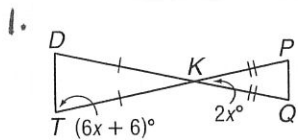
Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

Converse of the Isosceles Triangle Theorem (4.10):

Corollaries

4.3	4.4
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Practice: Find x

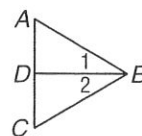


7. Write a two-column proof.

Given: $\triangle ABC$ is equilateral; $\angle 1 \cong \angle 2$.

Prove: $\angle ADB \cong \angle CDB$

Proof:



Statements

Reasons

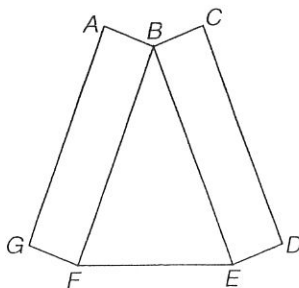
4-6 Enrichment

Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

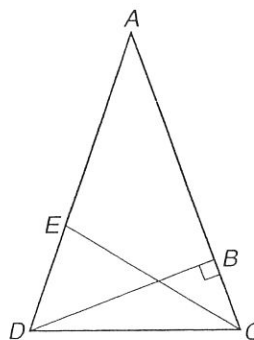
1. Given: $BE = BF$, $\angle BFG \cong \angle BEF \cong \angle BED$, $m\angle BFE = 82$ and $ABFG$ and $BCDE$ each have opposite sides parallel and congruent.

Find $m\angle ABC$.



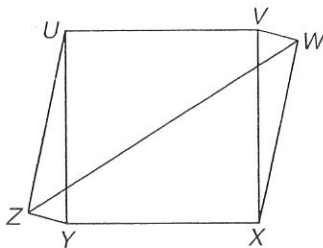
2. Given: $AC = AD$, and $\overline{AB} \perp \overline{BD}$, $m\angle DAC = 44$ and \overline{CE} bisects $\angle ACD$.

Find $m\angle DEC$.



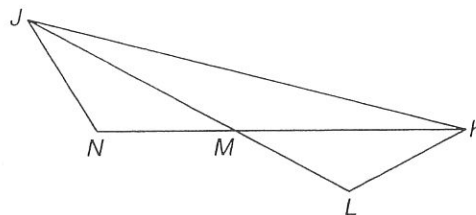
3. Given: $m\angle UZY = 90$, $m\angle ZWX = 45$, $\triangle YZU \cong \triangle VWX$, $UVXY$ is a square (all sides congruent, all angles right angles).

Find $m\angle WZY$.



4. Given: $m\angle N = 120$, $\overline{JN} \cong \overline{MN}$, $\triangle JNM \cong \triangle KLM$.

Find $m\angle JKM$.



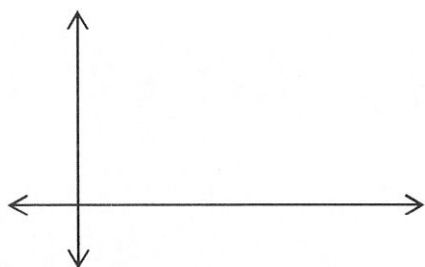
4-7 Triangles and Coordinate Proof

Coordinate proof:

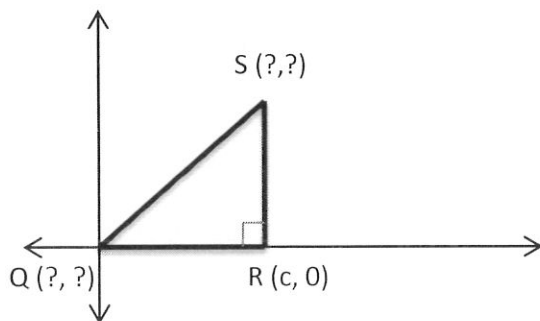
Placing Figures on the Coordinate Plane:

1.
2.
3.
4.

Example 1: Position and label isosceles triangle JKL on a coordinate plane so that base \overline{JK} is a units long.



Example 2: Name the missing coordinates of isosceles right $\triangle QRS$.



Write Coordinate Proofs: We can use coordinate proof to verify properties and to prove theorems.

The most common formulas used in coordinate proof:

- 1.
- 2.
- 3.

Example 3: Coordinate Proof

a: Write a coordinate proof to prove that the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

Step 1: Place the figure on the coordinate plane.

Step 2: Label the vertices and use variables for unknown coordinates.

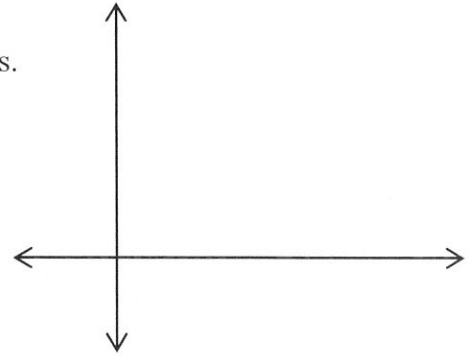
(Hint: Use multiples of 2 when using the Midpoint Formula)

Step 3: Write the given and what you need to prove.

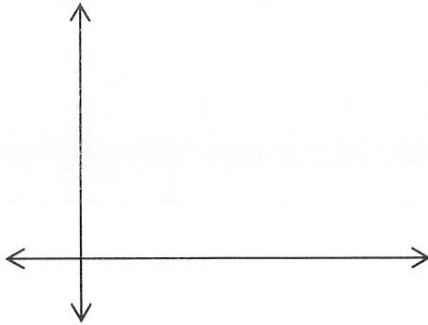
Step 4: Prove using a formula.

Given:

Prove:

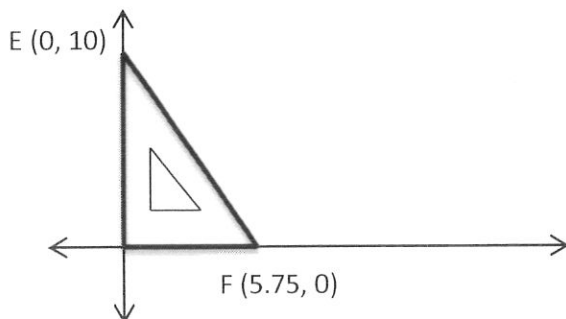


b: Write a coordinate proof to prove that the segment that joins the vertex angle of an isosceles triangle to the midpoint of its base is perpendicular to the base.



Example 4: Classify Triangles

Write a coordinate proof to prove that this drafter's tool is shaped like a right triangle. The length of one side is 10 inches and the length of another side is 5.75 inches.



Geometry Chapter 4 Notes:

1) CPCTC	Corresponding Parts of Congruent Triangles are Congruent
2) Angle Sum Thm.	Interior angles of a Δ add up to 180° .
3) 3rd Angle Thm.	If 2 angles of 2 triangles are \cong , then the 3 rd angles are \cong .
4) Exterior Angle Thm.	Exterior angles of a Δ are = the sum of the remote interior angles.
5) Δ Congruence	Reflexive, Symmetric, and Transitive Properties of \cong apply to Δ 's.
6) SSS Post.	2 Δ 's are \cong if all 3 of their sides are \cong .
7) SAS Post.	2 Δ 's are \cong if 2 sides and the included angle are \cong .
8) ASA Post.	2 Δ 's are \cong if 2 angles and the included side are \cong .
9) AAS Thm.	2 Δ 's are \cong if 2 angles and the non-included side are \cong .
10) Hypothesis Leg Postulate	2 right Δ 's are \cong if hypotenuse and corresponding leg are \cong .
11) Isosceles Δ Thm.	If a Δ has 2 \cong sides, its opposite angles are \cong .
12) Conv. Isosceles Δ Thm.	If a Δ has 2 \cong angles, its opposite sides are \cong .
13) Corollary 4.1	The acute angles of a right Δ are complementary.
14) Corollary 4.2	A Δ can have at most 1 right/obtuse angle.
15) Corollary 4.3	An equilateral Δ is always equiangular, and vice versa.
16) Corollary 4.4	Each angle of an equiangular Δ is 60° .