### **Honors Geometry Chapter 4 Packet**

#### 4-1 Classifying Triangles

- Identify and classify triangles by angles and sides.

#### 4-2 Angles of Triangles

- Apply the Angle Sum Theorem.
- Apply the Exterior Angle Theorem.

### 4-3 Congruent Triangles

- Name and label corresponding parts of congruent triangles.
- Identify congruence transformations.

#### 4-4 Proving Congruence-SSS, SAS

- Use the SSS Postulate to test for triangle congruence.
- Use the SAS Postulate to test for triangle congruence.

#### 4-5 Proving Congruence- ASA, AAS

- Use the ASA Postulate to test for triangle congruence.
- Use the AAS Theorem to test for triangle congruence.
- Use the HL-Postulate to test for right triangle congruence.

#### 4-6 Isosceles Triangles

- Use properties of isosceles triangles.
- Use properties of equilateral triangles.

#### 4-7 Triangles and Coordinate Proof

- Position and label triangles for use in coordinate proofs.
- Write coordinate proofs.

<sup>\*</sup>Remember all postulates, theorems, and formulas from prior chapters.

## 4-1 Classifying Triangles

### **Classifying Triangles by Angles**

Acute

Right

Obtuse

Equiangular

#### Classifying Triangles by Sides

Scalene

Isosceles

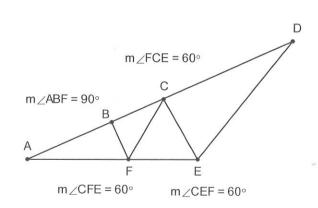
Equilateral

### Answer with always, sometimes or never.

An isosceles triangle is an equilateral triangle.

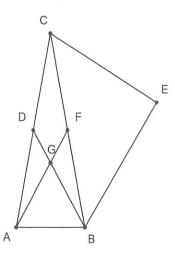
An equilateral triangle is an isosceles triangle.

**Ex 1:** The triangular truss below is modeled for steel construction. Classify  $\triangle ADE$ ,  $\triangle ABF$ , and  $\triangle FCE$  by angles.



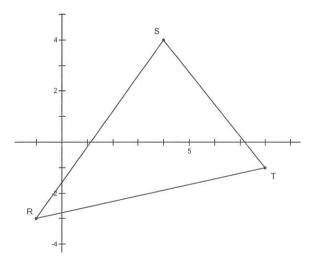
Is  $\overline{BF} \parallel \overline{CE}$ ? Justify your answer.

**Ex 2:** Identify each triangle in the figure by sides if  $\overline{CE} \perp \overline{BE}$ ,  $\overline{AG} \cong \overline{GB}$ ,  $\overline{AC} \cong \overline{CB}$ .



**Ex 3:** Find *d* and the measure of each side of the equilateral  $\Delta KLM$  if KL = d + 2, LM = 12 - d, and KM = 4d - 13.

**Ex 4:** Find the measures of the sides of  $\Delta RST$ . Classify the triangle by sides.



Turn to page 184 and complete the Geometry Activity. Answer the corresponding questions below.

1.

7.

- 2.
- 3.

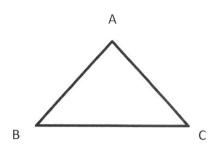
12.

- 4.
- 5.
- 6.

HW: pgs. 181-182 #28-31, 38, 40

## **4-2 Angles of Triangles**

### Angle Sum Theorem (4.1):

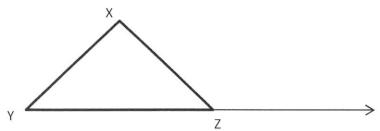


### **Proof of the Angle Sum Theorem**

	Statement	Reason
1.		1.
2.		2.
3.		3.
4.		4.
5.		5.
6.		6.
7.	The form I distance in the contract of	7.
8.		8.
9.		9.
10.		10.

Third Angle Theorem (4.2):

### **Exterior Angle:**



**Remote Interior Angles:** 

**Exterior Angle Theorem (4.3):** 

Corollary:

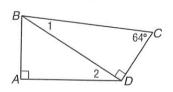
Corollary 4.1:

Corollary 4.2:

1. Find the measure of each angle if  $\angle BAD$  and  $\angle BDC$  are right angles and  $m\angle ABC = 84$ .

 $m \angle 1$ 

 $m \angle 2$ 



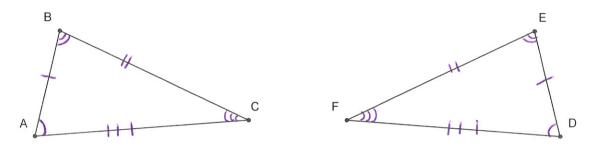
**2. CONSTRUCTION** The diagram shows an example of the Pratt Truss used in bridge construction. Use the diagram to find  $m \angle 1$ .

### 4-3 Congruent triangles

**Congruent Triangles:** 

**Definition of congruent Triangles (CPCTC):** 

#### **CPCTC** stands for:



$$\triangle ABC \cong \triangle DEF$$

\* The corresponding vertices can be used to name the corresponding congruent sides and angles of the two triangles.

$$\angle A \cong \angle O$$
,  $\angle B \cong \angle E$ ,  $\angle C \cong \angle F$   
 $\overline{AB} \cong \overline{DE}$ ,  $\overline{BC} \cong \overline{EF}$ ,  $\overline{CA} \cong \overline{FD}$ 

Thm 4.4: Congruence of triangles is also reflexive symmetric, and transitive.

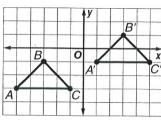
Symmetric	Transitive
	·
1	
*	
	Symmetric

<sup>\*</sup> Congruent sides are opposite congruent angles.

### **Congruence Transformations:**

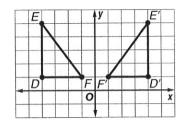
Slide

$$\triangle ABC \cong \triangle A'B'C'$$



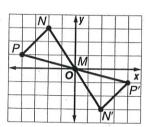
Flip

$$\triangle DEF \cong \triangle D'E'F'$$



Turn

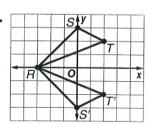
AMNP=AMN'P'



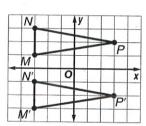
### Exercises

Describe the congruence transformation between the two triangles as a *slide*, a *flip*, or a *turn*. Then name the congruent triangles.

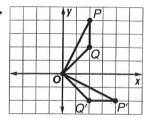
1.



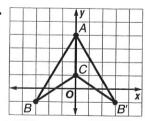
2.



3.

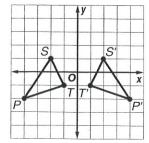


4.

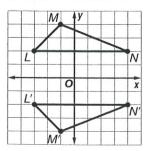


Verify that each of the following transformations preserves congruence, and name the congruence transformation.

**5.** 
$$\triangle PST \cong \triangle P'S'T'$$



**6.** 
$$\triangle LMN \cong \triangle L'M'N'$$



### 4-4 Proving Congruence- SSS, SAS

Warm-up: Currently, how can we prove triangles congruent?

SSS Postulate:

Example Write a two-column proof.

Given:  $\overline{AB} \cong \overline{DB}$  and C is the midpoint of  $\overline{AD}$ .

Prove:  $\triangle ABC \cong \triangle DBC$ 



Statement	Reason

### Exercises

Write a two-column proof.

1.

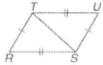




Given:  $\overline{AB} \cong \overline{XY}, \overline{AC} \cong \overline{XZ}, \overline{BC} \cong \overline{YZ}$ 

Prove:  $\triangle ABC \cong \triangle XYZ$ 

State	ements	Reasons	
*************************		A CONTRACT OF THE PARTY OF THE	-



Given:  $\overline{RS} \cong \overline{UT}, \overline{RT} \cong \overline{US}$ 

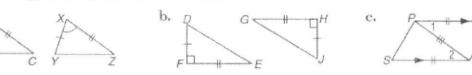
Prove:  $\triangle RST = \triangle UTS$ 

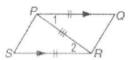
atements	Reasons	Statements	Reasons
			p. m
			And the second s
			meridani

#### **SAS Postulate:**

Example For each diagram, determine which pairs of triangles can be proved congruent by the SAS Postulate.







#### Exercises

For each figure, determine which pairs of triangles can be proved congruent by the SAS Postulate.













4-4

### **Skills Practice**

### Proving Congruence—SSS, SAS

Determine whether  $\triangle ABC \cong \triangle KLM$  given the coordinates of the vertices. Explain.

1. 
$$A(-3, 3), B(-1, 3), C(-3, 1), K(1, 4), L(3, 4), M(1, 6)$$

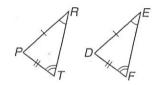
**2.** 
$$A(-4, -2)$$
,  $B(-4, 1)$ ,  $C(-1, -1)$ ,  $K(0, -2)$ ,  $L(0, 1)$ ,  $M(4, 1)$ 

3. Write a flow proof.

Given:  $\overline{PR} \cong \overline{DE}, \overline{PT} \cong \overline{DF}$ 

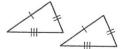
 $\angle R \cong \angle E, \angle T \cong \angle F$ 

**Prove:**  $\triangle PRT \cong \triangle DEF$ 



Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

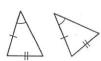
4.



5.



6.



### 4-4

### **Practice**

### Proving Congruence—SSS, SAS

Determine whether  $\triangle DEF \cong \triangle PQR$  given the coordinates of the vertices. Explain.

**1.** 
$$D(-6, 1), E(1, 2), F(-1, -4), P(0, 5), Q(7, 6), R(5, 0)$$

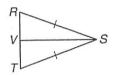
**2.** 
$$D(-7, -3)$$
,  $E(-4, -1)$ ,  $F(-2, -5)$ ,  $P(2, -2)$ ,  $Q(5, -4)$ ,  $R(0, -5)$ 

3. Write a flow proof.

Given:  $\overline{RS}\cong \overline{TS}$ 

V is the midpoint of  $\overline{RT}$ .

**Prove:**  $\triangle RSV \cong \triangle TSV$ 



Determine which postulate can be used to prove that the triangles are congruent. If it is not possible to prove that they are congruent, write *not possible*.

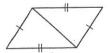
4.



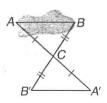
5.



6.



**7. INDIRECT MEASUREMENT** To measure the width of a sinkhole on his property, Harmon marked off congruent triangles as shown in the diagram. How does he know that the lengths *A'B'* and *AB* are equal?



### 4-5 Proving Congruence – ASA, AAS

#### Included side:

**Angle-Side-Angle Congruence Postulate (ASA post):** 

Angle-Angle-Side Congruence Theorem (AAS thm):

### **Proof of AAS Theorem:**

Given:

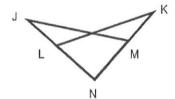
**Proof:** 

Statement	Reason
1.	1.
2.	2.
3.	3.

### Write a proof:

Given:  $\angle NKL \cong \angle NJM$ ,  $\overline{KL} \cong \overline{JM}$ 

Prove:  $\overline{LN} \cong \overline{MN}$ 



Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.

### 4-5

### **Practice**

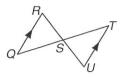
### Proving Congruence—ASA, AAS

1. Write a two column proof.

Given: S is the midpoint of  $\overline{QT}$ .

 $\overline{QR} \parallel \overline{TU}$ 

**Prove:**  $\triangle QSR \cong \triangle TSU$ 

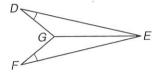


2. Write a two column proof.

Given:  $\angle D \cong \angle F$ 

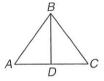
 $\overline{GE}$  bisects  $\angle DEF$ .

**Prove:**  $\overline{DG} \cong \overline{FG}$ 



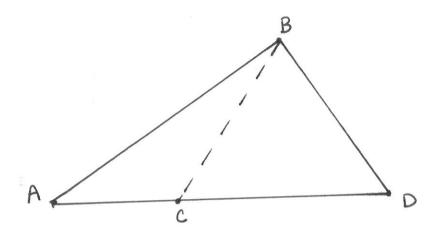
## ARCHITECTURE For Exercises 3 and 4, use the following information.

An architect used the window design in the diagram when remodeling an art studio.  $\overline{AB}$  and  $\overline{CB}$  each measure 3 feet.



- **3.** Suppose D is the midpoint of  $\overline{AC}$ . Determine whether  $\triangle ABD \cong \triangle CBD$ . Justify your answer.
- **4.** Suppose  $\angle A \cong \angle C$ . Determine whether  $\triangle ABD \cong \triangle CBD$ . Justify your answer.

## 4-5 Proving Congruence- ASS



Given:

**Prove:** 

$$\triangle$$
 ABC  $\cong$   $\triangle$  ABD

Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.

**ASS Congruence Postulate:** 

### 4-5 Congruence in Right Triangles Activity pgs. 214 to 215

Goal: Complete Activities 1 and 2 by answering the questions 1-6 in groups. Compare the Right Triangle Congruence theorems and postulates to the theorems and postulates we have already learned. Identify one we have not learned and record it in your notebook.

Challenge: Write a two-column proof for #10 &11

#### **Activity 1:**

2a. 3. 1a. b. b. C. C.

#### **Activity 2:**

Step 1: XY = 7cm4. Step 2: YZ ⊥ XY 5. Step 3: XZ = 10 cm6.

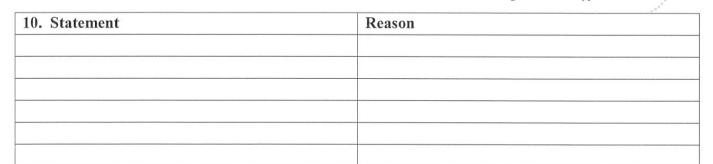
Hypotenuse-Leg Congruence (HL Post): If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

Use the figure to write a two-column proof.

10.	Given:	$\overline{ML}$	T	$\overline{MK}$ ,	JΚ	1	$\overline{KM}$	
-----	--------	-----------------	---	-------------------	----	---	-----------------	--

11. Given:  $\overline{JK} \perp \overline{KM}$ ,  $\overline{JM} \cong \overline{KL}$  $\angle I \cong \angle L$  $\overline{ML} \parallel \overline{IK}$ 

Prove:  $\overline{IM} \cong \overline{KL}$ Prove:  $\overline{ML} \cong \overline{IK}$ 

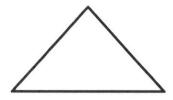


11. Statement	Reason

### **Summary of Methods to Prove Triangle Congruence**

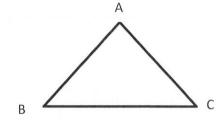
## **4-6 Isosceles Triangles**

**Vertex Angle:** 



**Base Angles:** 

**Isosceles Triangle Theorem (4.9):** 



**Proof of the Isosceles Triangle Theorem** 

Given:  $\overline{AB} \cong \overline{AC}$ 

Prove:

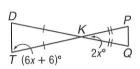
Statement	Reason
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.

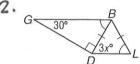
Converse of the Isosceles Triangle Theorem (4.10):

### Corollaries

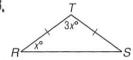
4.3	4.4	

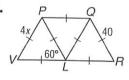
Practice: Find x



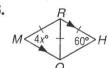


3.









7. Write a two-column proof.

**Given:**  $\triangle ABC$  is equilateral;  $\angle 1 \cong \angle 2$ .

**Prove:**  $\angle ADB \cong \angle CDB$ 



**Proof:** 

**Statements** 

Reasons

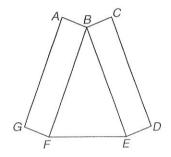
### **Enrichment**

### Triangle Challenges

Some problems include diagrams. If you are not sure how to solve the problem, begin by using the given information. Find the measures of as many angles as you can, writing each measure on the diagram. This may give you more clues to the solution.

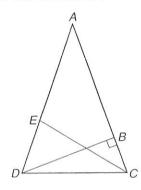
**1.** Given: BE = BF,  $\angle BFG \cong \angle BEF \cong$  $\angle BED$ ,  $m \angle BFE = 82$  and ABFG and BCDE each have opposite sides parallel and congruent.

Find  $m \angle ABC$ .



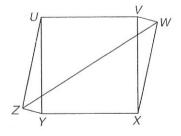
**2.** Given: AC = AD, and  $\overline{AB} \perp \overline{BD}$ ,  $m \angle DAC = 44$  and  $\overline{CE}$  bisects  $\angle ACD$ .

Find  $m \angle DEC$ .

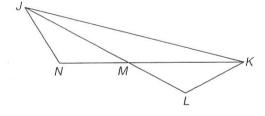


3. Given:  $m \angle UZY = 90$ ,  $m \angle ZWX = 45$ ,  $\triangle YZU \cong \triangle VWX$ , UVXY is a square (all sides congruent, all angles right angles).

Find  $m \angle WZY$ .



**4.** Given:  $m \angle N = 120$ ,  $\overline{JN} \cong \overline{MN}$ ,  $\triangle JNM \cong \triangle KLM$ . Find  $m \angle JKM$ .



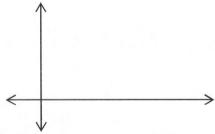
## **4-7 Triangles and Coordinate Proof**

### **Coordinate proof:**

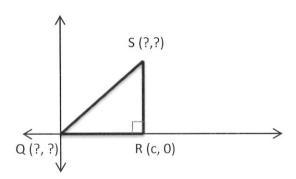
Placing Figures on the Coordinate Plane:

1.		
2.		
3.		
4.		

**Example 1:** Position and label isosceles triangle JKL on a coordinate plane so that base  $\overline{JK}$  is a units long.



**Example 2:** Name the missing coordinates of isosceles right  $\triangle QRS$ .



Write Coordinate Proofs: We can use coordinate proof to verify properties and to prove theorems.

The most common formulas used in coordinate proof:

1.

2.

3.

### **Example 3: Coordinate Proof**

**a:** Write a coordinate proof to prove that the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

**Step 1:** Place the figure on the coordinate plane.

**Step 2:** Label the vertices and use variables for unknown coordinates.

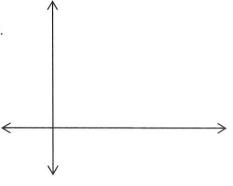
(Hint: Use multiples of 2 when using the Midpoint Formula)

Step 3: Write the given and what you need to prove.

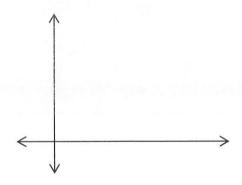
Step 4: Prove using a formula.

Given:

Prove:

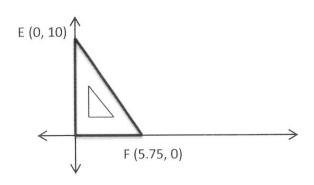


**b**: Write a coordinate proof to prove that the segment that joints the vertex angle of an isosceles triangle to the midpoint of its base is perpendicular to the base.



Example 4: Classify Triangles

Write a coordinate proof to prove that this drafter's tool is shaped like a right triangle. The length of one side is 10 inches and the length of another side is 5.75 inches.



Name:	Period:
Chapter 4: Congruent Triangles	Date: 1/19/12
Part 1: True or False	
1. In a right triangle more than one a	ingle can be right.
2. An isosceles triangle can be equila isosceles.	ateral but an equilateral triangle cannot be
	rior angles are greater than its exterior angle. wed using a theorem is called a corollary.
5. Triangles that are different sizes b	out the same shape are congruent.
6. The three congruence transforma	tions are slide, flip, and turn.
7. The angle made by two lines that	share a common vertex is an included angle.
8. Four methods to prove triangle co	ongruence are SAS, SSS, ASS, and AAS.
9. If two angles in a triangle are cong	gruent, the triangle is isosceles.
10. If the sides of one triangle are co triangles are congruent.	ngruent to the sides of another triangle, then the
Part 2: Fill in the Blank	arozopii aramo
concepts.	coordinate plane and algebra to prove geometric sides of a isosceles triangle is called the
<ul><li>3. The side between two given angles i</li><li>4. The interior angles of the triangle no</li></ul>	s called the ot adjacent to a given exterior angle are called
5. Three ways to classify a triangle by i	ts sides are,, and
Part 3: Multiple Choice	
1. How many acute angles must an acua.1 b. 2 c.3	te triangle have? d.0
2. Which postulate states that if the hyp congruent to the hypotenuse and leg of congruent?	
a. Leg-leg $\cong$ b. Hypotenuse angle $\cong$	c. Leg-angle $\cong$ d. Hypotenuse-leg $\cong$
3. How many obtuse angles must an obta. 1 b. 2 c. 3	

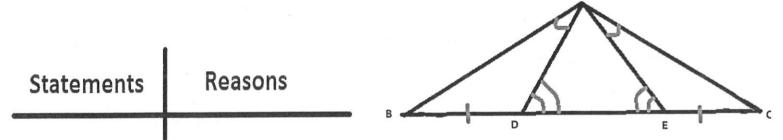
Name:	Period:
Chapter 4: Congruent Triangles	Date: 1/19/12

#### Part 4: Short Answer

1. What are 4 methods to prove triangle congruence and give each ones explanation.

Part 5: Use a Two-Column Proof to Answer the Question in the Space Provided.

1. Given:  $\langle BAD \cong \langle CAE, \langle ADE \cong \langle AED, Side BD \cong Side CE$ Prove: Triangle BAD  $\cong$  Triangle CAE



# Geometry Chapter 4 Notes:

1)CPCTC()-	Corresponding Parts of Congruent
f -M	Triangles are Congruent
2)Angle Sum Thm.	Interior angles of a ∆ add up to 180°.
	1.1
3)3rd Angle Thm.	If 2 angles of 2 triangles are $\cong$ , then the
X	$\beta^{rd}$ angles are $\cong$ .
4)Exterior Angle Thm.	Exterior angles of a $\triangle$ are = the sum of
	the remote interior angles.
5)Δ Congruence	Reflexive, Symmetric, and Transitive
	Properties of $\cong$ apply to $\triangle$ 's.
6)SSS Post.	$2\Delta$ 's are $\cong$ if all 3 of their sides are $\cong$ .
7)SAS Post.	$2\Delta$ 's are $\cong$ if 2 sides and the included
. ,5	angle are $\cong$ .
8)ASA Post.	$2\Delta$ 's are $\cong$ if 2 angles and the included
1. 10 C.MI	side are ≅.
9)AAS Thm.	$2 \Delta$ 's are $\cong$ if 2 angles and the non-
	included side are $\cong$ .
10) Hypothesis Leg Postulate	2 right ∆'s are ≅ if hypotenuse and
	corresponding leg are ≅.
11)Isosceles Δ Thm.	If a $\triangle$ has $2 \cong$ sides, its opposite angles
	are ≅.
12)Conv. Isosceles ∆ Thm.	If a $\triangle$ has $2 \cong$ angles, its opposite sides
	are ≅.
13)Corollary 4.1	The acute angles of a right ∆ are
7	complementary.
14)Corollary 4.2	$A \triangle can have at most 1 right/obtuse$
	angle.
15)Corollary 4.3	An equilateral ∆ is always equiangular,
112	and vice versa.
16)Corollary 4.4	Each angle of an equiangular ∆ is 60°.
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å<sup>th</sup> Grade Challenge Geometry